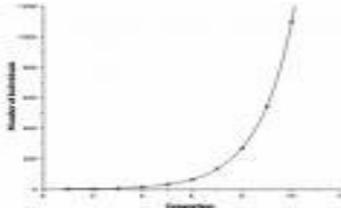


| | |
|---|--|
| <p>See a bracket – deal with it first!!</p> $5(x + 2) = 15$ $5x + 10 = 15$ $5x = 5$ $x = 1$ | <p>To simplify surds like $\sqrt{50}$, you can split a surd into its factors</p> <p>When splitting, look for square numbers</p> $\sqrt{50} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$ $\sqrt{98} = \sqrt{49} \times \sqrt{2} = 7\sqrt{2}$ $\sqrt{50} + \sqrt{98} = 5\sqrt{2} + 7\sqrt{2} = 12\sqrt{2}$ |
| <p>If a sequence is</p> <p>First term – 2×3 Second term – 3×4 Third term – 4×5 Fourth term = 5×6 LOOK FOR A PATTERN!! Then nth term = $(n+1)(n+2)$</p> | <p>To find a % increase, you must do</p> <p>% increase = increase/original x 100</p> <p>e.g. To increase from 7 to 12</p> $\% \text{ increase} = 5 / 7 \times 100 = 71\%$ |
| <p>To find equation of a straight line use $y = mx + c$</p> <p>$m = \text{gradient}$</p> <p>$c = y \text{ intercept}$</p> | <p>To increase by 5%, multiply by 1.05</p> <p>To decrease by 17% multiply by 0.83</p> |
| <p>If a surd is on the bottom of a fraction, get rid of it!!</p> <p>If a surd is on the bottom such as $\sqrt{37}$, then multiply top and bottom by $\sqrt{37}$</p> $\frac{7}{\sqrt{5}} = \frac{7\sqrt{5}}{5}$ | <p>If given that after a 22% decrease something costs £5</p> <p>then</p> <p>we know 78% = £5,</p> <p>then we find 1% = 6.41p</p> <p>and then 100% to get</p> <p>original price = £6.41</p> |

| | |
|---|---|
| <p>Difference of two squares</p> $x^2 - 25 = (x + 5)(x - 5)$ $4x^2 - 25 = (2x + 5)(2x - 5)$ | <p>Difference of two squares</p> $53^2 - 47^2 = (53+47)(53-47) = 600$ $95^2 - 5^2 = (95+5)(95-5) = 9000$ |
| <p>Solve inequalities like equations but remember the sign at the end!</p> <p>Only change sign if multiplying/dividing by a negative!</p> | <p>When substituting follow the laws of BIDMAS ie Brackets Indices (powers) Division/ Multiplication Addition/ Subtraction</p> $\frac{1}{2} y^2 \text{ when } y = 4 \text{ is } \frac{1}{2} \text{ of } 4^2 = \frac{1}{2} \text{ of } 16 = 8$ |
| <p>To rationalise a denominator, multiply top and bottom by the surd on the bottom!</p> $\frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{\sqrt{5}\sqrt{5}} = \frac{3\sqrt{5}}{5}$ | <p>A negative power means 1 over what it would be if it was positive! A posh word for one over is reciprocal.</p> $2^{-2} = \frac{1}{4}$ $5^{-3} = \frac{1}{125}$ |
| <p>When we raise a power to a power, multiply the powers!</p> $(x^2)^3 = x^6$ <p>If number inside, put that to power outside also!</p> $(3x^4)^2 = 9x^8$ | <p>Power $\frac{1}{2}$ means square root</p> <p>Power $\frac{1}{3}$ means cube root</p> |
| <p>You can't add surds like this: $\sqrt{2} + \sqrt{3} = \sqrt{5}$</p> <p>Only add if they're the same: $\sqrt{5} + \sqrt{5} = 2\sqrt{5}$</p> | <p>Power $\frac{2}{3}$ means cube root then square</p> <p>Power $\frac{3}{4}$ means do fourth root then cube</p> |

| | |
|--|---|
| <p>To add/ subtract fractions, get a common denominator</p> $\frac{3}{5} + \frac{7}{15} = \frac{9}{15} + \frac{7}{15} = \frac{16}{15} = 1\frac{1}{15}$ $\frac{3}{5} - \frac{7}{15} = \frac{9}{15} - \frac{7}{15} = \frac{2}{15}$ | <p>Before calculating with fractions turn any mixed fraction into an improper one!</p> $1\frac{3}{5} \div \frac{2}{7} = \frac{8}{5} \times \frac{7}{2} = \frac{56}{10} = 5\frac{6}{10} = 5\frac{3}{5}$ |
| <p>If using quadratic formula beware of negatives!</p> | <p>Never use factorisation or quadratic formula to solve until what you want to solve equals zero!</p> |
| <p>Solve $x^2 + y^2 = 10, x + y = 4$ But $y = 4 - x$ so substitute linear into quadratic</p> $x^2 + (4 - x)^2 = 10$ $x^2 + 16 - 8x + x^2 = 10$ $2x^2 - 8x + 6 = 0$ $x^2 - 4x + 3 = 0$ $(x - 1)(x - 3) = 0$ $x = 1, 3, y = 3, 1$ | <p>IDENTITY – \equiv 3 lines</p> <p>EXPRESSION – no equality sign</p> <p>FORMULA – more than one unknown, would need information about at least one variable to find others,</p> <p>EQUATION – one unknown that can be solved</p> |
| <p>If a line has gradient 2, its perpendicular has gradient $-\frac{1}{2}$</p> <p>If a line has gradient $-\frac{1}{4}$, its perpendicular has gradient 4</p> <p>The gradient product of two perpendicular lines is -1</p> |  <p>$y = 2^x$ looks like this, as does $y = \text{anything to the power } x$ DRAW IT ACCURATELY BY PLOTTING POINTS!!</p> |

When working with algebra use brackets to stop silly mistakes

Always factorise or cancel when you can!!

On a number line a filled in circle means \geq or \leq

On a number line an open circle means $>$ or $<$

Compound interest is accumulated interest.

If a bank account has £1000 and interest is 9% then after 5 years the account will have $1000 \times 1.09 \times 1.09 \times 1.09 \times 1.09 \times 1.09$ or 1000×1.09^5

When rearranging

Use brackets

Cancel if you can

Factorise if you can

Always, when making something the subject, make sure you have it alone on one side only.

To do this, make sure you start with the intention of getting everything with your subject in together!!

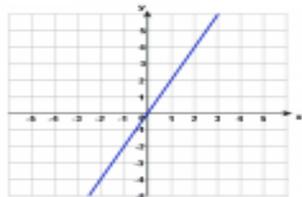
Gradient = up over across

Standard form-

If a number isn't in standard form rewrite first part in standard form as below

$$0.000000000967 \times 10^{14} = 9.67 \times 10^{-10} \times 10^{14} = 9.67 \times 10^4$$

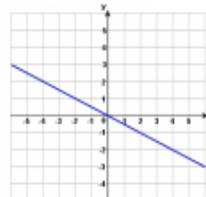
A LINE WITH POSITIVE GRADIENT LOOKS LIKE



THIS!

Eg $y=2x+3$

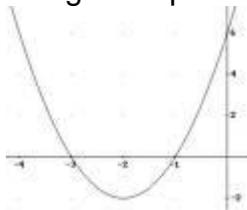
A LINE WITH NEGATIVE GRADIENT LOOKS LIKE THIS!



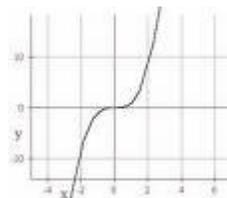
Eg $y=-x$, $y=-4x-5$

A QUADRATIC GRAPH SUCH AS $y = x^2$ IS U SHAPED.

If we have a negative quantity of x^2 the graph is



n shaped.



A CUBIC GRAPH SUCH AS $y = x^3$ IS SHAPED AS ABOVE.

To multiply fractions, multiply top by top and bottom by bottom

$$\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$$

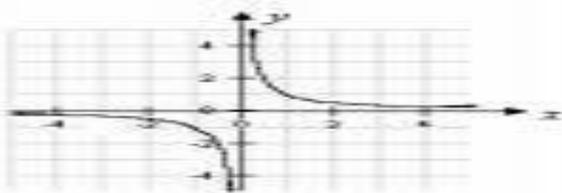
To divide fractions, flip the second fraction and multiply

$$\frac{3}{5} \div \frac{2}{3} = \frac{3}{5} \times \frac{3}{2} = \frac{9}{10}$$

When using mixed fractions, convert into improper fractions first

$$1\frac{3}{4} \div 3\frac{2}{7} = \frac{7}{4} \div \frac{23}{7} = \frac{7}{4} \times \frac{7}{23} = \frac{49}{92}$$

A reciprocal graph looks like this eg $\frac{1}{x}$, $\frac{5}{6x}$



Draw it accurately by PLOTTING POINTS!!

With trial and improvement, when you know the answer is between two 1 decimal place values check the middle value to ensure which of the values is closer.

SOLVE $x^2 - 3x = 7$ to 1 dp

| x | $x^2 - 3x$ | comment |
|------|------------|----------|
| 4 | 4 | Too low |
| 5 | 10 | Too high |
| 4.5 | 6.75 | Too low |
| 4.6 | 7.36 | Too high |
| 4.55 | 7.0525 | Too high |

So answer lies between 4.5 and 4.55 so is 4.5 to 1dp

When we divide powers of the same number or variable we subtract the powers

$$x^8 \div x^5 = x^3$$

$$6x^7 \div 18x^9 = \frac{1}{3}x^{-2}$$

When we multiply powers of the same number or variable we add the powers

$$x^2 \times x^5 = x^7$$

$$3y^8 \times 5y^2 = 15y^{10}$$

| | |
|---|--|
| <p>In proof, let any integer be n</p> <p>Then $2n$ will always be even</p> <p>And $2n+1$ will always be odd</p> | <p>Prove that the sum of the square of an odd number and the square of an even number is 1 more than a multiple of 4</p> <p>Odd=$2n+1$ even = $2n$ Sum of squares = $(2n)^2 + (2n+1)^2 = 4n^2+4n^2+4n+1 = 8n^2+4n+1=4(2n^2+n)+1$</p> <p>As the 4 has been factorised out then the answer is 1 more than a multiple of 4</p> |
| <p>To solve a quadratic equation, use the following methods in order of preference</p> <p>Factorise Use the quadratic formula Complete the square</p> <p>Before trying to solve, get it equal to zero!</p> | <p>To factorise an expression look for the HIGHEST common factor of each term and put this outside the bracket. Inside the bracket must go what you multiply the highest factor by to get each term.</p> <p>$5x - 15 = 5(x - 3)$ $15x^3y^5 - 6x^2y^3 = 3x^2y^3(5xy^2 - 2)$</p> |
| <p>If there is no common factor when factorising, look for two brackets.</p> <p>$x^2 + 7x + 12 = (x + 3)(x + 4)$</p> <p>The two numbers in the brackets add to get the middle number in the expression and multiply to get the end number.</p> <p>$x^2 - 5x + 4 = (x - 1)(x - 4)$ $x^2 - 36 = (x - 6)(x + 6)$</p> | <p>To find the nth term of a linear sequence (goes up by the same amount each time)</p> <p>nth term = (common difference) \times n + zero term</p> <p>3,8,13,18 nth term = $5n - 2$ 90, 74, 58, 42 nth term = $-16n + 106$</p> |
| <p>Never cancel until you are multiplying everything top and bottom or until you have factorised!</p> $\frac{x^2 + 5x}{x^2 + 6x + 5} = \frac{x(x+5)}{(x+1)(x+5)} = \frac{x}{x+1}$ | <p>Expand $(\sqrt{7} + 3)(\sqrt{5} - 4)$</p> <p>Use FOIL to get</p> $\sqrt{35} - 4\sqrt{7} + 3\sqrt{5} - 12$ |
| <p>Is a triangle with sides $\sqrt{3} + 1$, $\sqrt{3} - 1$ and $\sqrt{8}$ right angled?</p> <p>Well, does Pythagoras work?</p> $(\sqrt{3} + 1)^2 = 3 + 2\sqrt{3} + 1 = 4 + 2\sqrt{3}$ $(\sqrt{3} - 1)^2 = 3 - 2\sqrt{3} + 1 = 4 - 2\sqrt{3}$ $(\sqrt{3} + 1)^2 + (\sqrt{3} - 1)^2 = 8$ so hypotenuse would be $\sqrt{8}$ so it is right angled | <p>Using Completing the square 1: Solving Quadratics</p> $x^2 + 6x - 7 = 0$ $(x + 3)^2 - 9 - 7 = 0$ $(x + 3)^2 - 16 = 0$ $(x + 3)^2 = 16$ $x + 3 = \pm 4$ $x = -7, 1$ |

**Using Completing the square 2:
minimum**

What is minimum of $x^2 + 6x - 7$?
Complete the square to get $(x+3)^2 - 16$
Then minimum is -16 because the expression
above is minimum when square is zero
So minimum is -16 when $x=-3$

In probability, if a question asks for who has the most accurate results then it is always the person who has performed more trials

**Before solving a quadratic
equation make it equal to zero**

$$8^{\frac{2}{3}} = 2^2 = 4$$
$$27^{-\frac{2}{3}} = \frac{1}{27^{\frac{2}{3}}} = \frac{1}{9}$$
$$\left(\frac{8}{27}\right)^{\frac{2}{3}} = \frac{8^{\frac{2}{3}}}{27^{\frac{2}{3}}} = \frac{4}{9}$$

If in any doubt

Factorise

or use **Pythagoras**

When comparing distributions always comment on the average and spread

e.g. Boys marks on average are better but the range is higher so the marks are more spread out; they are less consistent

**The gradient product of two
perpendicular lines is -1**

Give the equation of the perpendicular to $y=2x$
at (0,3)

Gradient of perpendicular = $-\frac{1}{2}$ Y intercept = 3

So equation is $y = -\frac{1}{2}x + 3$

Parallel lines have the same gradient

Give the equation of the line passing through (2,6)
which is parallel to $y=2x-3$

Gradient of parallel = 2

So equation is $y=2x + \text{something}$

Because it passes through (2,6), when $x=2$, $y=6$

So $6=4 + \text{something}$ so something = 2

Equation is $y=2x+2$

When y is directly proportional to x the rule
 $y = kx$ exists

When y is directly proportional to x^2 the
rule $y = kx^2$ exists

When y is inversely proportional to x the rule $y = \frac{k}{x}$ exists.

When y is inversely proportional to x^2 the rule $y = \frac{k}{x^2}$ exists.

| | |
|---|--|
| <p>Example: y is directly proportional to x^2. When $x=2$, $y=8$. What is y when $x = 7$</p> <p>The rule $y = kx^2$ exists. So $8 = 4k$, $k=2$</p> <p>When $x =7$, $y = 2x^2$, $y= 2 \times 49= 98$</p> | <p>Example: y is inversely proportional to x^2. When $x=2$, $y=1$. What is y when $x = 2$?</p> <p style="text-align: right;">The rule $y = \frac{k}{x^2}$ exists</p> <p>So $1=k/4$ so $k=4$</p> <p>When $x =2$, $y = 4/x$, $y= 2$.</p> |
| <p>Turning Recurring decimals into fractions</p> $x = 0.423232323232323$ $100x = 42.32323232323232$ $99x = 41.9$ $x = \frac{41.9}{99} = \frac{419}{990}$ | <p>When turning recurring decimals into fractions,</p> <p>Multiply by 10 if one repeating decimal</p> <p>Multiply by 100 if two repeating decimals</p> <p>Multiply by 1000 if three repeating decimals</p> |
| <p>To find a recurring decimal just divide</p> $\frac{4}{9} = 4 \div 9 = 0.44444444444444$ | <p>If a question requires you to solve. but doesn't give an equation, use your initiative</p> <p>Call whatever you start with x and work from there</p> |
| <p>Solve</p> $\frac{3}{x-1} - \frac{2}{x+1} = 1$ $3 - \frac{2(x-1)}{x+1} = x-1$ $3(x+1) - 2(x-1) = (x-1)(x+1)$ $3x+3 - 2x+2 = x^2 - 1$ $x^2 - x - 6 = 0$ $(x-3)(x+2) = 0$ $x = -2, 3$ | <p>3.7^{05} ON A CALCULATOR MEANS 370000</p> |
| <p>If a question asks you to put an expression in the form $(x + a)^2 + b$, this means</p> <p style="text-align: center;">COMPLETE THE SQUARE!</p> | <p style="text-align: center;">Reciprocal means 1 over!!</p> <p>So the reciprocal of 5 is $1/5$</p> <p>The reciprocal of $2/3$ is $3/2$ (turn the fraction upside down)</p> |

| | |
|---|---|
| <p>If asked for an irrational number use pi or a square root</p> <p>e.g. an irrational number between 5 and 6 is</p> <p>$\pi - 1$ OR the square root of 26 or 27 or 28 or 29 or 30 or 31 or 32 or 33 or 34 or 35 or 36</p> | <p>If a sequence is non linear, it must have something to do with n^2, so take the sequence 1,4,9,16,25 off the original sequence and see what's left</p> <p>So 2,3,6,11, 18</p> <p>Take off 1,4,9,16,25 to get 1, -1, -3, -5, -7</p> <p>The nth term of this is $3 - 2n$</p> <p>So the whole sequence is made up of $n^2 - 2n + 3$</p> |
| <div style="text-align: center;"> $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ </div> <p>remember that you are dividing the whole thing by 2a!!!</p> <p>a= number in front of x^2</p> <p>b= number in front of x</p> <p>c= constant</p> | <p>Solve $2x^2 - 5x - 6 = 0$ using the quadratic formula.</p> <div style="background-color: #ffffcc; padding: 10px;"> $x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-6)}}{2(2)}$ $= \frac{5 \pm \sqrt{25 + 48}}{4}$ $= \frac{5 \pm \sqrt{73}}{4}$ $= \frac{5 \pm 8.544}{4}$ </div> <p>So $x = \frac{5 + 8.544}{4}$ or $x = \frac{5 - 8.544}{4}$</p> $= \frac{13.544}{4}$ $= 3.386$ $= 3.39 \text{ correct to 2dp}$ $= \frac{-3.544}{4}$ $= -0.886$ $= -0.89 \text{ correct to 2dp}$ |
| <p>In SHOW THAT questions,</p> <p>Full answers are required-</p> <p>STATE THE OBVIOUS!</p> <p>Don't miss out any steps!!</p> | <p>When estimating, a rule of thumb is to round all quantities to 1 SIGNIFICANT FIGURE.</p> <p>Sometimes 2sf is better but it would be made pretty obvious.</p> <p>e.g. $\frac{39.1}{0.77} \approx \frac{40}{0.8} \approx \frac{400}{8} \approx 50$</p> |
| <p>Don't get Highest common factor and least common multiple mixed up!!</p> <p>e.g. 72 and 108</p> <p>HCF = 36</p> <p>LCM = 216</p> | <p>To write a number as a product of its prime factors, do as below!!</p> <p>$108 = 2^2 \times 3^3$ $72 = 2^3 \times 3^2$</p> |

$$108 = 2^2 \times 3^3$$

$$72 = 2^3 \times 3^2$$

You can find HCF and LCM using these but
IT IS NOT RECOMMENDED.

HCF – highest power of all common numbers used
 $= 2^2 \times 3^2 = 36$

LCM = highest power of all common numbers
used
 $= 2^3 \times 3^3 = 216$

Graphical inequalities

When testing which side to shade, test a point on the appropriate side of the line before you shade!!

Don't round until the end of a question.

Try to use full answers throughout your calculations,
then round appropriately at the end!!

Be careful when adding quantities with standard form

DO NOT ADD THE POWERS!

e.g. $7.5 \times 10^3 + 8.2 \times 10^2 = 7500 + 820 = 8320 = 8.32 \times 10^3$

Simplify $\frac{1}{x} + \frac{3}{2-x}$

Get a common denominator

$$\frac{2-x}{x(2-x)} + \frac{3x}{x(2-x)} = \frac{2x+2}{2x-x^2}$$

You may be asked to use the quadratic formula without a calculator!!

Solve

$$3x^2 + 5x - 1 = 0$$

a=3, b = 5, c = -1

$$\text{So } x = \frac{-5 \pm \sqrt{5^2 - (4 \times 3 \times -1)}}{2 \times 3} = \frac{-5 \pm \sqrt{37}}{6}$$

To work out what percentage 230 is of 1460 do

$$\frac{230}{1460} \times 100 = 15.8\%$$

Upper/Lower bounds example

Find max speed of Sue's journey if she travelled 25 miles (2sf) in 2.1 hours (1dp)

$$\begin{aligned}\text{Max speed} &= \text{max distance} / \text{min time} \\ &= 25.5 / 2.05 = 12.4 \text{ mph}\end{aligned}$$

Suitable degree of accuracy means

To same accuracy as quantities in the question OR

To 3 significant figures

FACTORISE $6x^2 + 25x - 9$

$ac = -54$, find two factors of this which add to b

These are 27 and -2

$$\begin{aligned}\text{Rewrite as } 6x^2 + 27x - 2x - 9 \\ 3x(2x+9) - 1(2x+9)\end{aligned}$$

Factorise in 2 parts $(3x-1)(2x+9)$

$$(\sqrt{5}+3)(\sqrt{5}-3) = 5 - 3\sqrt{5} + 3\sqrt{5} - 9 = -4$$

Notice how multiplying expressions which differ only in the sign in the middle, the answer is RATIONAL!

So, we can say that

$$\frac{1}{\sqrt{5}+3} = \frac{\sqrt{5}-3}{(\sqrt{5}+3)(\sqrt{5}-3)} = \frac{\sqrt{5}-3}{-4} = \frac{3-\sqrt{5}}{4}$$

Solving:

- Factorising
- Formula
- Completing the square
- Drawing a graph

Factorising:

easy ... $x^2 + 7x + 12 = 0$
 $(x + 3)(x + 4) = 0$
 $x = -3$ or $x = -4$

brackets

... more difficult!

multiply

$3x^2 - 5x + 2$
 $3x^2 - 3x - 2x + 2$
 $3x(x - 1) - 2(x - 1)$
 $(3x - 2)(x - 1)$

6
 $\frac{1 \times 6}{2 \times 3}$

Quadratic Equations

$ax^2 + bx + c$

Completing the square:

$$x^2 + 4x - 3 = 0$$

$$(x + 2)^2 - 4 - 3 = 0$$

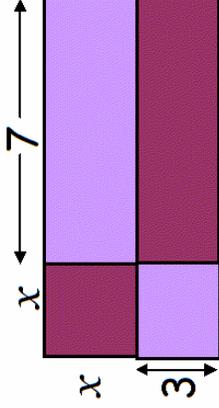
$$(x + 2)^2 - 7 = 0$$

$$x + 2 = \pm\sqrt{7}$$

$$x = \pm\sqrt{7} - 2$$

half of 4x

subtract 2²



Difference of Two Squares:

$$x^2 - 16$$

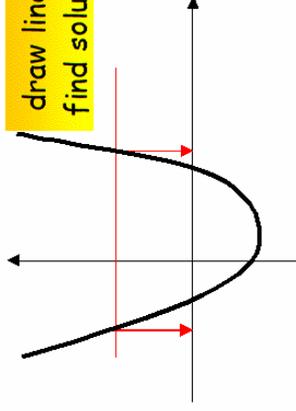
$$(x - 4)(x + 4)$$

x squared subtract 4 squared

The formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Graphs:



Parabola - u shaped graph